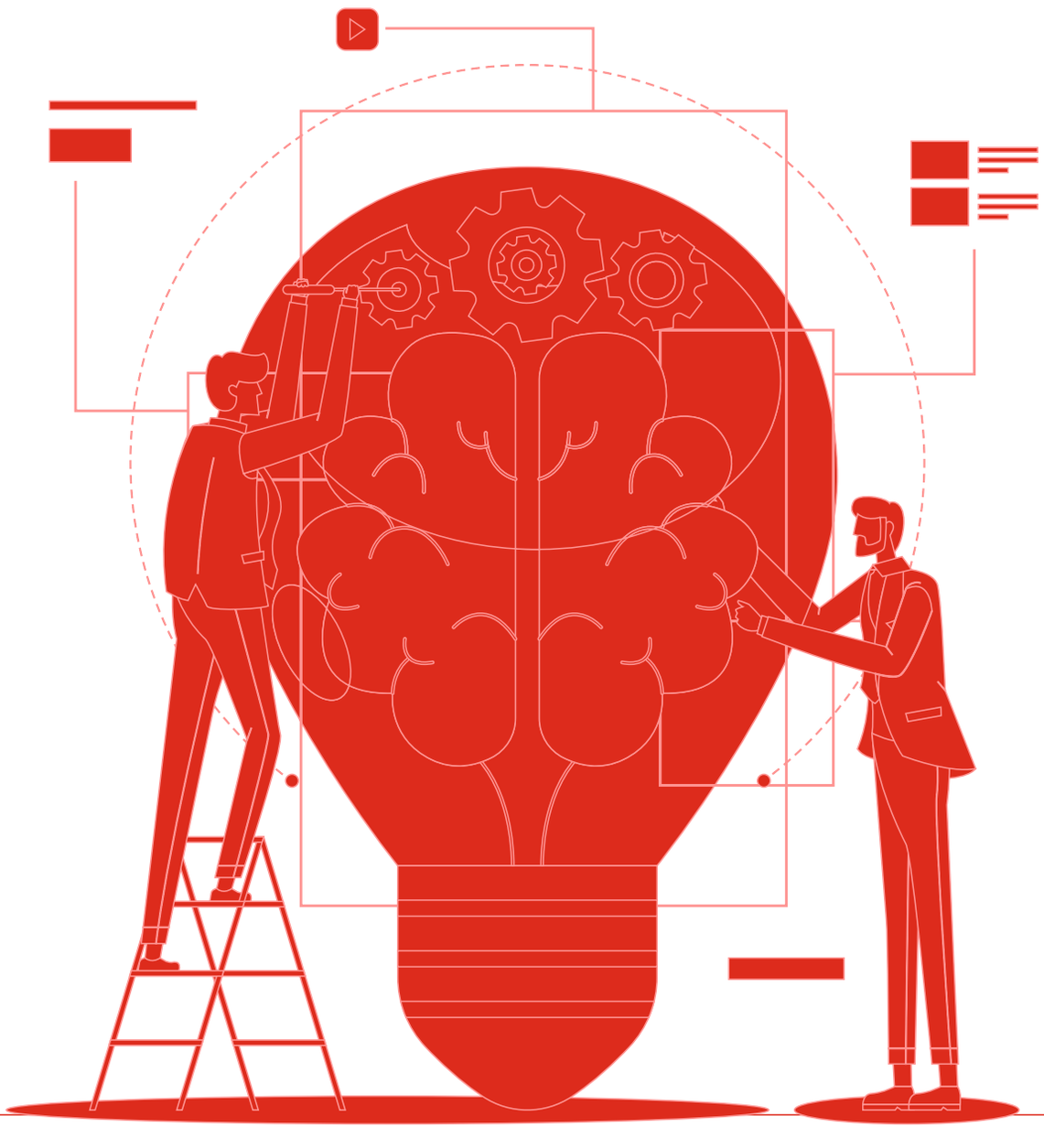
ACTION

is the real measure of INTELLIGENCE

- CA VINOD REDDY -



YOUR INTELLIGENCE makes you really ATTRACTIVE

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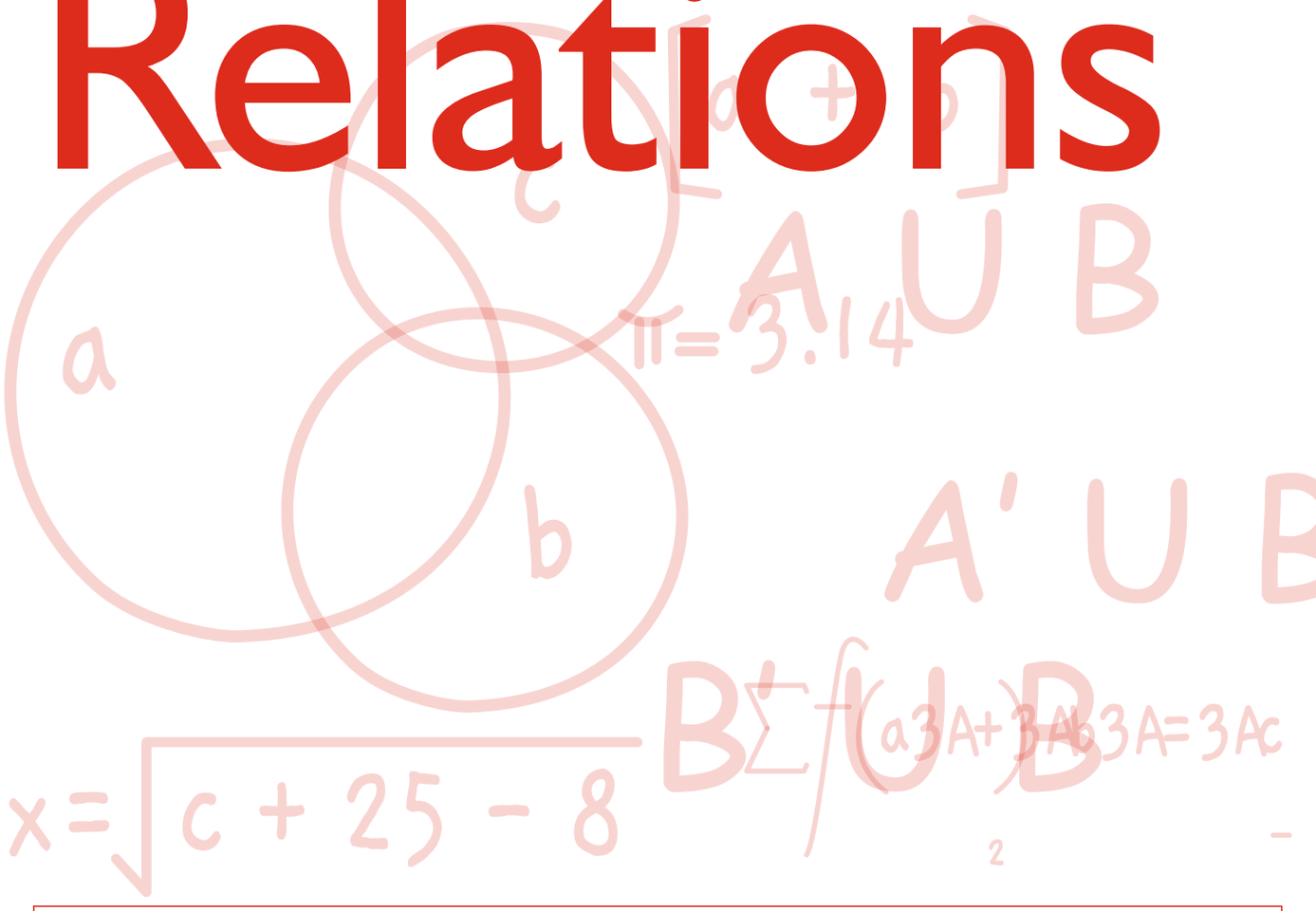
Sets, Functions, Relations



CA VINOD REDDY

1 Set is a collection of well defined and Distinct objects

- Roster Form / Braces Form / List form
  - A = {1,2,3,4,5,6,7,8,9,10}
- Algebric Form / Rule Form / Property Form / Set Builder Form / Description form
  - A is a set of first 10 natural numbers
  - OR
  - A = {x : where x ∈ N and x ≤ 10}

2 In mathematics everything in this world whether living or non-living, is called as an object or Elements

3 A = {5,8,9,10,13} Explain : ∈

5 ∈ A : 5 belongs to set A

10 ∈ A : 10 belongs to set A

200 ∉ A : 200 does not belong to set A.

13 ∈ A : 13 belongs to set A

m ∉ A : 'm' does not belong to set A

B = {Accounts, maths, Law, Law, Law, maths, Accounts, Eco, Eco, Eco, Eco, Eco}

4 No. of distinct elements of a set is known as cardinal value ↓ n(B) = 4

5 Types of sets on the basis of elements

- Null set OR Empty set OR void set = ϕ OR { }
  - ↓ cardinal value is 'zero'
- singleton set
  - ↓ cardinal value is '1'
- finite set
  - ↓ cardinal value is finite / Limited / countable
- Infinite set
  - ↓ cardinal value is infinite / unlimited / uncountable

6
- Generally name of the set is denoted by capital letters.
- Order of object is Not Relevant
- Repetition is of no use

A = {1,2,3,4,5} B = {5,4,4,5,1,2,2,3,4,5,4}

sets A and B are name sets

7 Equivalent sets : $A = \{5,8,9,13\}$ $B = \{YR, P, M, 383\}$

Here $n(A) = 4$ & $n(B) = 4$

$\therefore$ A, B are Equivalent sets

If cardinal value of 2 or more sets is same then sets are said to be equivalent sets.

8 Subset : $A = \{2,3,5,8\}$ $B = \{1,2,3,4,5,6,7,8,9,10\}$

Here Every obsh of set A belongs to set B also

$\therefore$ A is a subset of B or B is a superset of A.

9 Superset : $A = \{a,b,c\}$, $B = \{a, b, c, d, e\}$ birdie $D = \{a,b,c\}$

A is a subset of B : A is a proper subset of B.

D is a subset of B : D is a proper subset of B

A is a subset of D : A is an improper subset of D

10 Proper Subset : $A = \{5,10\}$ $B = \{5,10,18\}$

A is a proper subset of B $\Rightarrow A \subset B$

11 Improper Subset :

$X = \{10,18,20\}$, $Y = \{18,20,10\}$

X is a subset but not a proper subset of Y

i.e. X is improper subset of Y.

My Notes

Find All possible subsets of $A = \{10,20,30\}$

$\Rightarrow$

$\{10\}, \{20\}, \{30\}, \{10,20\}$

$\{10,30\}, \{20,30\}, \phi$

$\{10,20,30\}$

$\left.\begin{matrix}\{10\}, \{20\}, \{30\}, \{10,20\} \\ \{10,30\}, \{20,30\}, \phi\end{matrix}\right]$ proper subsets

$\{10,20,30\}$] improper subset

12 Find all possible subsets of A = {5,7,8}

Proper subsets : {5}, {7}, {8}, {7,5}, {5,8}, {7,8}, ϕ

Improper subset : {5,7,8}

13 For set B = {a,b,c}

All possible subsets : $2^3 = 8$

All possible proper subsets : $2^3 - 1 = 7$

All possible improper subsets : 1

All possible empty subsets : 1

All possible non-empty subsets : $2^3 - 1 = 7$

All possible Non-empty proper subsets $= 2^3 - 2 = 6$

Find all subsets of {p} ⟹ {p}, ϕ

14 If cardinal value of a set = n; then

No. of subsets : 2^n

No. of proper subsets : $(2^n) - 1$

No. of improper subsets : 1

No. of empty subsets : 1

No. of non-empty subsets : $(2^n) - 1$

No. of non-empty proper subsets : $(2^n) - 2$

Find all subsets of {p, q} ⟹ {p}, {q}, ϕ, {pq} 4

Null set is a subset of any other set

15 When 2 sets are said to be equivalent sets?

① A = {3,4,8,9} B = {2,3,9,8}

Here A is a subset of B & B is a subset of A

∴ A, B are said to be Equal sets

As A, B are Etual sets, they are equivalent

also as n(A) = n(B) = 5

My Notes

② M = {2, 15, 100, Pim}, N = {a, b, P.m, a}

As n(M) = n(N) = 3

∴ M, N are Equivalent sets.

All Equal sets are Equivalent also but All Equivalent sets are not necessarily Equal sets

16 **When 2 or more sets are said to be equal sets?**

⟹ If $A \subseteq B$ & $B \subseteq A$ then

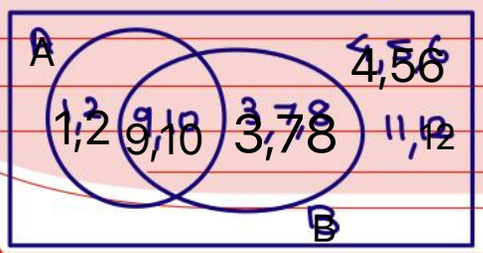
A, B are said to be Equal sets.

∴ All equal sets are equivalent but all equivalent sets are not necessarily equal sets.

17 **Universal Set :**

- A set of all observations under the scope of study is known as universal set.
- universal set is the superset of any other set
- universal set is denoted by 'U' or 'S'
- It is denoted as 'Rectangle' in venn diagram.

18 **Complementary Set :**



$A = \{4,3,9,10\}$

$A' = A^c$ = complementary set of A $= \{3,7,8,4,5,6,11,12\}$

$\therefore n(A) + n(A') = n(U)$

$B = \{9,10,3,7,8\}$

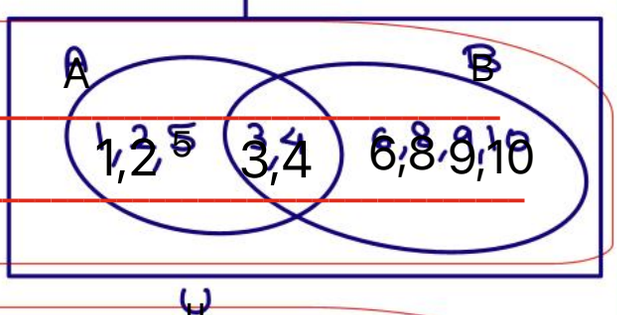
$B' = \{1,2,4,5,6,4,4256\}$

19 **If A = {1,2,3,4,5} B = {3,4,6,8,9,10}**

Find (A union B) = $\{1,3,5,3,4,6,8,9,10\} = (A \cup B)$

Find (A intersection B) = $\{3,4\} = (A \cap B)$

union = OR Intersection = and



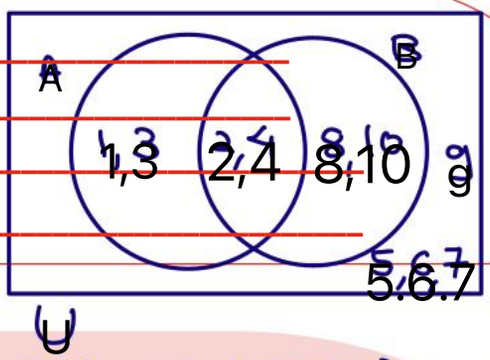
20 **If A = {1,2,3,4} B = {2,4,8,10} U = {1,2,3,4,5,6,7,8,9,10}**

Find A' = $\{5,6,7,8,9,10\}$

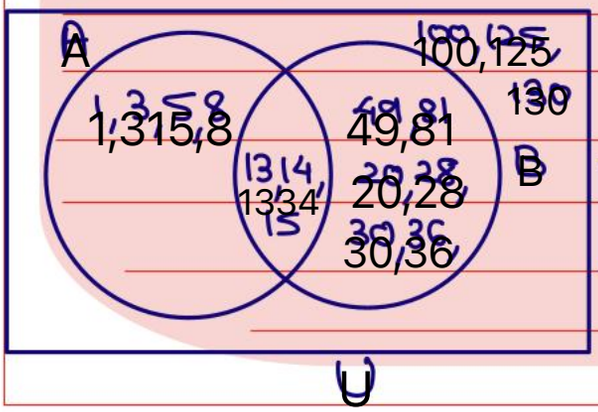
B' = $\{1,3,5,6,7,9\}$

(A ∪ B) = $\{1,2,3,4,8,10\}$

(A ∩ B) = $\{3,4\}$



My Notes

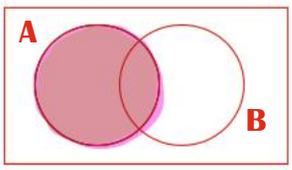


$n(A) = 7$, $n(B) = 9$, $n(A \cap B) = 3$

$n(A \cup B) = 13$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

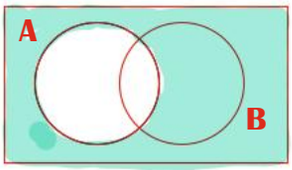
21 **Find Set A**



$n(A) = n(U) - n(A')$

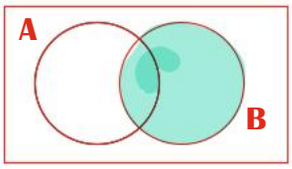
$n(A) + n(A') = n(U)$

22 **Find Set A'**



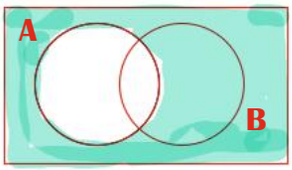
$n(A') = n(U) - n(A)$

23 **Find Set B**



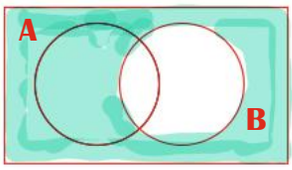
$n(B) = n(U) - n(B')$

24 **Find Set A'**



$n(A') = n(U) - n(A)$

25 **Find Set B'**



$n(B') = n(U) - n(B)$

My Notes

① $n(A) + n(A') = n(U)$

② $n(B') = n(U) - n(B)$

③ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

④ $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

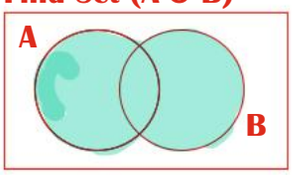
⑤ $n(A - B) = n(A \cap B') = n(A) - n(A \cap B)$

⑥ $n(B - A) = n(B \cap A') = n(B) - n(A \cap B)$

⑦ $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

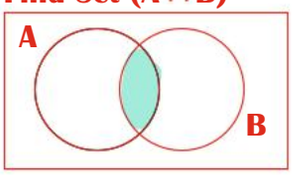
⑧ $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$

26 **Find Set (A ∪ B)**



$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

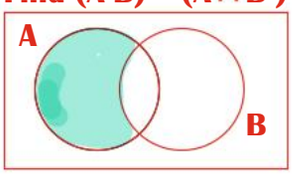
27 **Find Set (A ∩ B)**



$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

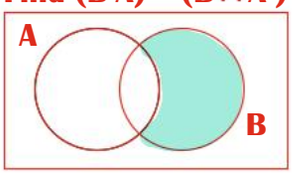
$n(A \cap B) = n(A) - n(A - B)$

28 **Find (A-B) = (A ∩ B')**



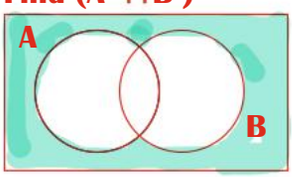
$n(A - B) = n(A \cap B') = n(A) - n(A \cap B)$

29 **Find (B-A) = (B ∩ A')**



$n(B - A) = n(B \cap A') = n(B) - n(A \cap B)$

30 **Find (A' ∩ B')**

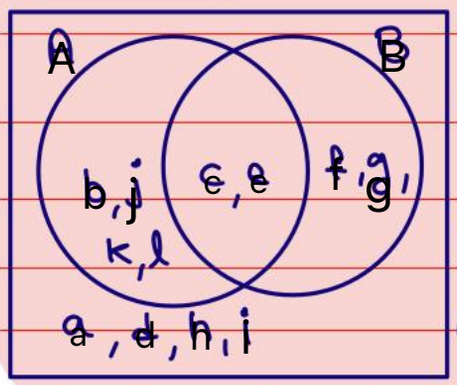


$n(A' \cap B') = n(A \cup B)'$

$= n(U) - n(A \cup B)$

De-morgan's rule on sets

My Notes



U = universal set

① $A = \{b, j, k, l, c, e\}$ $B = \{c, e, f, g\}$

③ $A' = \{f, g, a, d, h, i\}$ ④ $B' = \{b, j, k, l, a, d\}$

⑤ $A \cap B = \{c, e\}$ ⑥ $(A \cup B) = \{b, j, k, l, c, e, f, g\}$

⑦ $A - B = \{b, j, k, l\}$ ⑧ $B - A = \{f, g\}$

⑨ $(A' \cap B') = \{a, d, h, i\} = (A \cup B)'$

⑩ $A \Delta B = \{b, j, k, l, f, g\}$

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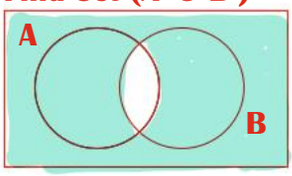
31 Find Set (A' ∪ B')

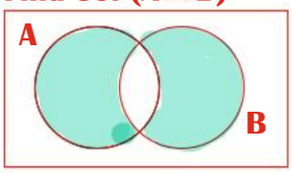


$n(A \cup B') = n(A \cap B)'$

$= n(U) - n(A \cap B)$

$(A' \cup B') = (A \cap B)'$ De Morgan's Rule onset

32 Find Set (A △ B)

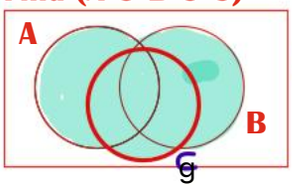


$(A \triangle B) = [(A - B) \cup (B - A)]$

$n(A \triangle B) = n(A - B) + n(B - A)$

$= n(A \cup B) - n(A \cap B)$

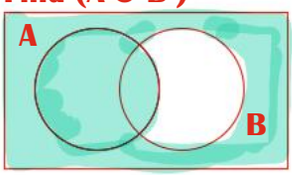
33 Find (A ∪ B ∪ C)



$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$

$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

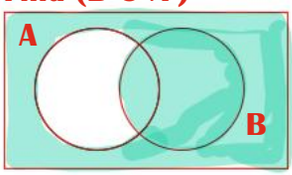
34 Find (A ∪ B')



$n(A \cup B') = n(U) - n(B - A)$

$= n(A) + n(A' \cap B')$

35 Find (B ∪ A')

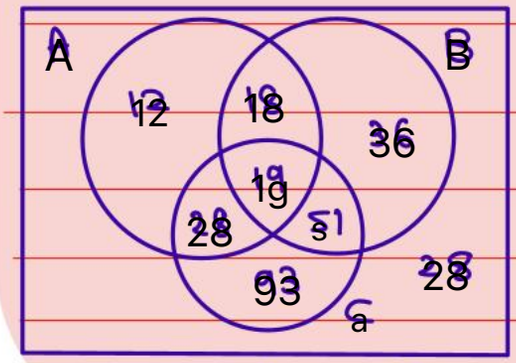


$n(B \cup A') = n(U) - n(A - B)$

$= n(B) + n(A' \cap B')$

$n(A \cup B \cup C) = n(U) - n(A' \cap B' \cap C')$

My Notes



U = universal set

$n(A) = 77$

$n(B) = 124$

$n(C) = 191$

$n(A \cap B) = 37$

$n(B \cap C) = 70$

$n(A \cap C) = 47$

$n(A \cap B \cap C) = 19$

$n(U) = 285$

$n(A \cup B \cup C) = 285 - 28 = 257$

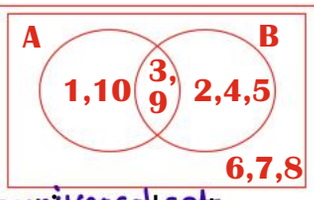
$= n(A) + n(B) + n(C) - n(A \cap B)$

$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$= 77 + 124 + 191 - 37 - 70 - 47 + 19$

$= 257$

36



U = universal set

Find A = { 1,10,3,9 }

Find B = { 3,9,2,4,5 }

Find A' = { 2,4,5,6,7,8 }

Find B' = { 1,10,6,7,8 }

Find A ∪ B = { 1,10,3,9,2,4,5 }

Find A ∩ B = { 3,9 }

Find A - B = { 1,10 }

Find B - A = { 2,4,5 }

Find A ∪ B' = { 1,10,3,9,6,7,8 }

Find A' ∩ B' = { 6,7,8 }

Find A' ∪ B' = { 1,10,2,4,5,6,7,8 }

Find B ∪ A' = { 3,9,1,2,4,5,6,7,8 }

37

Formulae of sets at one place

n(A') = n(U) − n(A)

n(B') = n(U) − n(B)

n(A∪B) = n(A) + n(B) − n(A∩B)

n(A∩B) = n(A) + n(B) − n(A∪B)

n(A - B) = n(A) − n(A∩B)

n(B - A) = n(B) − n(A∩B)

n(A'∩B') = n(A∪B)' = n(U) − n(A∪B)

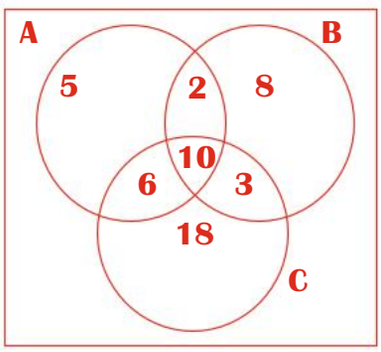
n(A △ B) = n(A - B) + n(B - A) = n(A∪B) − n(A∩B)

n(A'∪B') = n(A∩B)' = n(U) − n(A∩B)

n(A ∪ B') = n(B - A)' = n(U) − n(B - A)

n(B ∪ A') = n(A - B)' = n(U) − n(A - B)

38



n(A ∪ B ∪ C) = 52

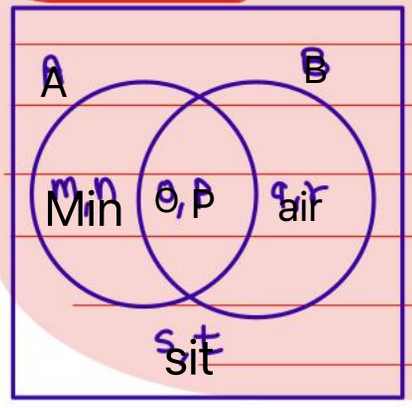
n(A∪B∪C) =

n(A) + n(B) + n(C) - n(A∩B) - n(A∩C) - n(B∩C) + n(A∩B∩C) =

= 23 + 23 + 37 − 12 − 16 − 13 + 10

= 52

My Notes



A = {m,n,o,p}

Be = {o,p,q,r}

A' = {q,r,s,t}

B' = {m,n,s,t}

A∩B = {o,p}

A∪B = {m,n,o,p,q,r}

A - B = {m,n}

B - A = {q,r}

A△B = {m,n,q,r}

(A'∩B') = {s,t}

(A'∪B') = {m,n,q,r,s,t}

39 Cartesian product of sets

If A = {1,2,3} B = {8,9}

Find (A x B) = {(1,8), (1,9), (2,8), (2,9), (3,8), (3,9)}

Find (B x A) = {(8,1), (8,2), (8,3), (9,1), (9,2), (9,3)}

$(A \times B) \neq (B \times A)$ but $n(A \times B) = n(B \times A) = 6$

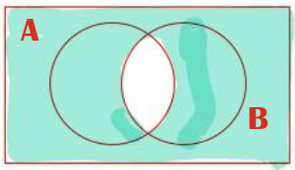
$(A \times B)$ & $(B \times A)$ are Equivalent sets but not equal sets.

40

A is a subset of B : Notation : $A \subseteq B$

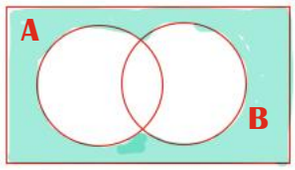
A is a proper subset of B : Notation : $A \subset B$

41 Demorgan's Rules of Sets



$(A' \cup B') = (A \cap B)'$

$n(A' \cup B') = n(U) - n(A \cap B)$



$(A \cup B)' = (A' \cap B')$

$n(A' \cap B') = n(U) - n(A \cup B)$

My Notes

If $n(A) = 537$, $n(B) = 1081$, $n(A \cap B) = 238$, $n(U) = 1980$

Find

⟹ ① $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 1380$

② $n(A \cap B') = n(A) - n(A \cap B) = 537 - 238 = 299$

③ $n(B \cap A') = n(B) - n(A \cap B) = 1081 - 238 = 843$

④ $n(A' \cap B') = n(U) - n(A \cup B) = 1980 - 1380 = 600$

⑤ $n(A \Delta B) = n(A \cap B') + n(B \cap A') = 299 + 843 = 1142$

⑥ $n(A' \cup B') = n(U) - n(A \cap B) = 1980 - 238 = 1742$

42

$A \cup A =$ A
$A \cap A =$ A
$A \cup \phi =$ A
$A \cap \phi =$ ϕ
$A \cup A' =$ U
$A \cap A' =$ ϕ
$A \cup U =$ U
$\phi' =$ U
$A \cup (A \cup B) =$ (AUB)
$(A \cup B) \cup (A \cap B) =$ (AUB)
$(A \cup B) \cup (A' \cap B') =$ U
$A \cup (A \triangle B) =$ (AUB)
$A \cup (A \cap B') =$ A
$(A \cap B') \cup (A \cap B) =$ A
$(A \triangle B) \cup (A \cap B) =$ (AUB)
$U' =$ ϕ

43

Any subject of the product set X.Y is said to define a relation from X to Y, and any relation from X to Y in which no 2 different ordered pairs have the same first element is called as function.

In $f : A \longrightarrow B$

the element f(x) of B is called as image of x while x is called as pre-image of f(x).

44

There are 4 types of relations

1. one to one
2. one to many
3. many to one
4. many to many

out of these 4 relations only
i) one to one } Relations are functions
ii) many to one
many

- Every function is a Relation but every Relation is not necessarily a function.

45

If $f(x) = 3x^2+2x+1$

Find f(3), f(8), f(-9), f(10)

$\Rightarrow f(x) = 3x^2 + 2x + 1$
$f(3) = 3(3)^2 + 2(3) + 1 = 34$
$f(-9) = 3(-9)^2 + 2(-9) + 1 = 226$
$f(10) = 3(10)^2 + 2(10) + 1 = 321$

when y is a function of x
then y = Dependent variable
x = Independent vasiable

4 $f(8) = 3(8)^2 + 2(8) + 1$
$= 209$

46

If f(x) = 8x+11; g(x) = 2x+9

Find f(3) = $8(3) + 11 = 35$
g(8) = $2(8) + 9 = 25$
g(p) = $2p + 9$
g(y) = $2y + 9$
f(-13) = $8(-13) + 11 = -93$
f(20) = $8(20) + 11 = 171$
g(2k) = $2(2k) + 9 = 4k + 9$

$f[g(10)] = f[2(10) + 9]$
$= f(29)$
$f \cdot g(10) = 8(29) + 11 = 243$
$g \cdot f(3) = g[8(3) + 11] = g(35)$
$= 2(35) + 9 = 79$

Function of Function is known as : composite function

47 If f(x) = 10x+15; g(x) = 7x - 13 **Find f.g(x), g.f(x)**

$\Rightarrow$ ① $f.g(x) = f(7x-13) = 10(7x-13) + 15$

$= 70x - 130 + 15 = 70x - 115$

② $g.f(x) = g(10x+15) = 7(10x+15) - 13$

$= 70x + 105 - 13 = 70x + 92$

48 If f(x) = 2x+11 **Find $f^{-1}(y)$, $f^{-1}(x)$, $f^{-1}(p)$**

$\Rightarrow y = f(x) = 2x + 11$

$y - 11 = 2x$

$x = \left(\frac{y-11}{2}\right)$

$f^{-1}(y) = \frac{y-11}{2}$

$f^{-1}(x) = \left(\frac{x-11}{2}\right)$

$f^{-1}(p) = \left(\frac{p-11}{2}\right)$

If Demand (y) is a function of price(x) then price is the Inverse function of demand

49 If $f(x) = \frac{2x+13}{8x-2}$; **Find $f^{-1}(y)$, $f^{-1}(20)$, $f^{-1}(p)$, $f^{-1}(p+1)$**

$\Rightarrow$

$f^{-1}(20) = \frac{2(20)+13}{8(20)-2} = \frac{53}{158}$

$f^{-1}(p) = \left(\frac{2p+13}{8p-2}\right)$

$f^{-1}(p+1) = \frac{2(p+1)+13}{8(p+1)-2} = \frac{2p+15}{8p+6}$

If $y = f(x)$ then $x = f^{-1}(y)$

$y = \frac{2x+13}{8x-2} = f(x)$

$8xy - 2y = 2x + 13$

$8xy - 2x = 2y + 13$

$x(8y-2) = 2y + 13$

$x = \left(\frac{2y+13}{8y-2}\right)$

$f^{-1}(y) = \left(\frac{2y+13}{8y-2}\right)$

50 If $f(x) = \frac{1}{1-x}$; **Find f(10), f(2), f(13), f(p), $f^{-1}(10)$, $f^{-1}(p)$**

$\Rightarrow y = f(x) = \frac{1}{1-x}$ $\therefore x = \frac{y-1}{y}$

$y - xy = 1$

$y - 1 = xy$

$f^{-1}(y) = \frac{y-1}{y}$

$f(10) = 1 - 1/10$

$f(2) = -1$

$f(13) = 1 - 1/12$

$f(p) = 1 - 1/p$

$f^{-1}(10) = 9/10$

$f^{-1}(p) = (p-1)/p$

51 If $g(x) = \frac{x-1}{x}$; **Find g(-1/2)**, $g^{-1}(y)$

$g(-\frac{1}{2}) = \frac{-\frac{1}{2}-1}{-\frac{1}{2}} = \frac{-1-2}{-1} = 3$

$y = g(x) = \frac{x-1}{x}$

$xy = x - 1$

$xy - x = -1$

$x(y-1) = -1$

$x = \frac{-1}{y-1} = \left(\frac{1}{1-y}\right)$

$\therefore g^{-1}(y) = \frac{1}{1-y}$

52 If f(2x+3) = 8x + 7. **Find f(x), f(30)**

$\Rightarrow f(2x+3) = 8x + 7$

$f(2x+3) = 4(2x+3) - 12 + 7$

$f(2x+3) = 4(2x+3) - 5$

$f(p) = 4p - 5$

$\therefore f(x) = 4x - 5$

$f(30) = 4(30) - 5 = 115$

If $f(x) = 4x - 5$ find $f(2x+3)$

$\Rightarrow f(2x+3) = 4(2x+3) - 5 = 8x + 7$

53 Domain and Range of {(1,5),(2,8),(3,9),(4,18)}

$\Rightarrow$ Domain = set of all first elements = {1,2,3,4}

Range = set of all second elements = {5,8,9,18}

54 f(x-1) = x^2. Find f(x), f(x+1)

$f(x-1) = x^2$

$f(x+1-1) = (x+1)^2$

$f(x) = (x+1)^2$

55 When a relation is said to be

Symmetric — If (a,b) is present in Relation then (b,a) should also be present

Reflexive — (a,a)

Transitive — If (a,b), (b,c) $\in$ R then (a,c) $\in$ R

56 Relation of Equivalence — If a Relation is Reflexive, symmetric, Transitive then it is said to be Relation of Equivalence.

57 'Is perpendicular to' is a symmetric Relation

58 'Is the reciprocal of' is a symmetric Relation

59 In a class of 100 students 60 like maths 50 like physics 25 like both subjects. Find how many students :

a. Like maths or physics = $n(A \cup B) = 85$

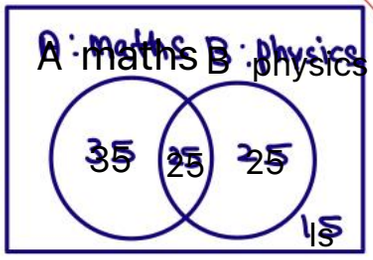
b. Like maths but not physics = $n(A - B) = n(A \cap B') = 35$

c. Like physics but not maths = $n(B - A) = n(B \cap A') = 25$

d. Neither like maths nor like physics = $n(A' \cap B') = 15$

e. Not like atleast one of 2 subjects = $n(A' \cup B') = n(A \cap B)' = 75$

f. Like one and only one subject = $n(A \Delta B) = 60$



A, B are said to be
- Equal sets if A = B i.e. A ⊆ B & B ⊆ A
- Equivalent sets if n(A) = n(B)

60 A = {5,8,9,10}; B = {8,5,9,10}; C = {a,b,c,d}

A, B are Equal Sets; Therefore Equivalent Also.

A, C are Equivalent Sets; but not Equal sets.

B, C are Equivalent sets but not equal sets .

61 Set of cubes of a natural numbers is _____ set

a. Finite ~~b. Infinite~~ c. Singleton d. Null

set of cubes of natural numbers = {1³, 2³, 3³, 4³ ----------}

62 {x : [(1) - (-1)^x]} for all integer values of x then x =

a. {0} b. {2} ~~c. {0,2}~~ d. None of these

$1-(-1)^x$; $x=1 \Rightarrow 1-(-1)^1 = 2$; $x=2 \Rightarrow 1-(-1)^2 = 0$; $x=3 \Rightarrow 1-(-1)^3 = 2$; $x=4 \Rightarrow 1-(-1)^4 = 0$

{30}

63 E is a set of all even natural numbers and O is a set of all odd natural numbers then

(E ∪ O) = {1,2,3,4,5,6,7,8 ----------} = N

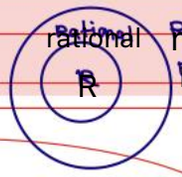
(E ∩ O) = ∅ = {0} = {2}

64 If R is a set of positive rational numbers and E is a set of all real numbers then

a. R ⊆ E ~~b. R ⊂ E~~ c. E ⊆ R d. E ⊂ R

All Rational nFbers are Real but all Real numbers are not necessarily rational.

∴ R is a proper subset of E.



65 If N is a set of all natural numbers and I is a set of positive integers then

~~a. N = I~~ b. N ⊂ I c. N ⊆ I d. I ⊆ N

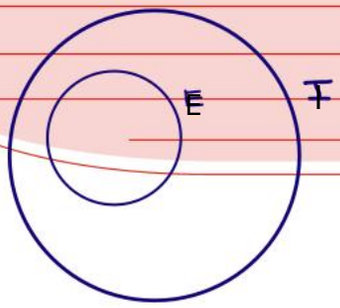
N = {1,2,3,4,5 -----}

I = {1,2,3,4,5 -----}

N, I are Equal sets

66 If I is a set of all isosceles triangles and E is a set of all equilateral triangles, then

a. I ⊂ E ~~b. E ⊂ I~~ c. E = I d. None of these



All equilateral triangles are isosceles triangles but All isosceles triangles are not necessarily equilateral triangles.

67 {[n(n+1)/2] where n is a positive integer} is a _____

a. Finite set ~~b. Infinite set~~ c. An empty set d. Singleton

68 If A = {1,2,3,4,5} B = {x^2 : x ∈ A} then -

a. n(A) > n(B) b. n(A) < n(B) ~~c. n(A) = n(B)~~ d. None

A = {1,2,3,4,5}

B = {1,4,9,16,25}

Here n(A) = n(B) = 5

69 Let f :A → B then A is called as domain of f, while B is called as co-domain of f.
Then set f(A) = {f(x) : x ∈ A} is called as

Range of f

A = {1,5,9,123} B = {1, 25, 81, 144, 169, 225}

f : A → B where f(x) = x^2

Domain = {1,5,9,123}

co-domain = {1,25,81,144,169,225}

Range = {1,25,81,144}

70 Let A = {1,2,3,4,5} B = {1,4,9,16,25,36,49}, we consider the rule f(x) = x^2

then f (1) = 1
f (2) = 4
f (3) = 9
f (4) = 16
f (5) = 25

Clearly each element of A has unique image in B

f : A → B : f(x) = x^2 is a function from A to B
where domain = {1,2,3,4,5}
Range = {1,4,9,16,25}

As in set A pre-image of 36,49 is not there it is 'INTO' function.

If each element of 'B' has atleast one pre-image in set A then function is said to be 'ONTO' function.

71 A one-one onto function is said to be bijective. A bijective function is also known as one to one correspondence.

Let f : A → B, defined in such a way that all elements in A have the same image in B, then f is said to be constant function

Two functions f and g are said to be equal written as f = g if they have the same domain and they satisfy the condition f(x) = g(x) for all values of x.

72 Inverse function is possible only when function is one to one onto

A = {Pune, Chennai} B = {MH, Bihar, TN}

A × B = {(Pune, MH), (Pune, Bihar), (Pune, TN), (Chennai, MH), (Chennai, Bihar), (Chennai, TN)}

Every subset of A × B

73 Inverse $h^{-1}(x)$ when $h(x) = \log_{10}x$ is :

a. $\log_{10}x$ b. 10^x c. $\log_{10}(1/x)$ d. None of these

74 For the function $h(x) = 10^{(1+x)}$ the domain of real values of x where $0 \le x \le 9$, the range is -

a. $10 \le h(x) \le 10^{10}$ b. $0 \le h(x) \le 10^{10}$ c. $0 \le h(x) \le 10$ d. None

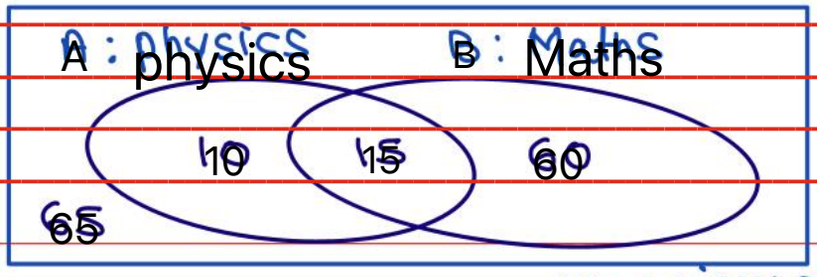
75 Let S = {a,b,c, ... } be any set then the relation R is a subset of the product set (S x S)

1. If R contains all ordered pairs (a,a) in (S x S) then R is said to be Reflexive
2. If (a,b) ∈ R, then (b,a) ∈ R. For every (a,b) ∈ S then R is said to be symmetric
3. If (a,b) ∈ R, and (b,c) ∈ R; then (a,c) ∈ R. For every a,b,c ∈ S then R is said to be Transitive

A relation which is reflexive symmetric as well as transitive is called as **Equivalence relation OR Relation of Equivalence**

76 In a class of 150 students 25 like physics, 75 like maths. 135 students dislike atleast one subject then find no. of students

1. Who like physics but not maths : $n(A-B) = n(A \cap B') = 10$
2. Who like maths but not physics : $n(B-A) = n(B \cap A') = 60$
3. Who like both subjects : $n(A \cap B) = 15$
4. Who like neither maths nor physics : $n(A' \cap B') = 65$
5. Who like one and only one subject : $n(A \Delta B) = 10 + 60 = 70$



77 "Is smaller than" over the set of eggs in a box is :

a. Transitive b. Symmetric c. Reflexive d. Equivalence

78 A = {2,3,8,9,11} B = {3,10,13} C = {5,10,13,15,19} Find A x (B∩C)

⟹ A = {2,3,8,9,11} B∩C = {10,13}

A x (B∩C) = {(2,10), (2,13), (3,10), (3,13), (8,10), (8,13), (9,10), (9,13), (11,10), (11,13)}

79 A = {2,8} B = {2,8} Find (AxB), (BxA), [(AxB) ∪ (BxA)], [(AxB)∩(BxA)]

A x B = {(2,2), (2,8), (8,2), (8,8)}

B x A = {(2,2), (2,8), (8,2), (8,8)}

(A x B) ∪ (B x A) = {(2,2), (2,8), (8,2), (8,8)}

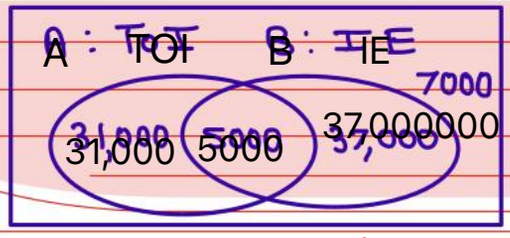
(A x B) ∩ (B x A) = {(2,2), (2,8), (8,2), (8,8)}

If A, B are Equal sets then

(A x B) = (B x A) = (A x B) ∪ (B x A)

= (A x B) ∩ (B x A)

80 A town has total population of 80,000. Out of it 36,000 read TOI, 42,000 read IE, 5000 read both, then find no. of persons who read one and only one newspaper?



U = universal set

n(A Δ B) = n(A − B) + n(B − A)

= 31,000 + 37,000 = 68,000

81 If f(x) = 1/(1-x) the $f^{-1}(x)$ = ?

a. (1-x) b. (x-1)/x c. x/(x-1) d. None of these

y = 1/(1−x)

y(1−x) = 1

y − xy = 1

y − 1 = xy

y = (y−1)/x

$\therefore f^{-1}(y) = \frac{y-1}{y}$

$\therefore f^{-1}(x) = \frac{x-1}{x}$

82 Null set is represented by

a. {ϕ} or 0 b. {} or ϕ c. ϕ or {0} d. None of these

Null set is also known as Empty set or void set which is denoted by: { } OR ϕ

My Notes

y = Vinod Reddy's Income

x = No. of students in his Batches

m = No. of students pursuing CA course in India

k = Demand of CA's in India

f = Industrial & service sector Growth rate of India

y = f(x)

y = f[g(m)]

y = f · g · h(k)

y = f · g · h · d(e)

83 If $f(x) = x^2$ then it is

a. Odd function b. Even function c. Both of these d. None of these

$\Rightarrow$ IF $f(x) = 22$

$f(-x) = (-a)^2 = 22$

Here $f(x) = f(-x)$ then $f(x)$ is said to be an Even function

84 f(x) is said to be an

Odd Function if $f(x) = -f(-x)$ or $-f(a) = f(-a)$

Even Function if $f(x) = f(-x)$

If $f(x) = 2^3 + 2^5$ then

$f(-x) = (-2)^3 + (-2)^5 = -2^3 - 2^5 = -(x^3 + x^5) = -f(x)$

$f(-x) = -f(x)$ Here $f(x)$ is an odd function.

85 If $f(x) = \text{Log}\left(\frac{1+x}{1-x}\right)$ then function f is said to be

a. Odd function b. Even function c. Both of these d. None of these

$f(x) = \text{Log}\left(\frac{1+x}{1-x}\right) = \text{Log}(1+x) - \text{Log}(1-x)$ ..... (1)

$f(-x) = \text{Log}\left(\frac{1-x}{1+x}\right) = \text{Log}(1-x) - \text{Log}(1+x) = -[\text{Log}(1+x) - \text{Log}(1-x)]$

$f(-x) = -f(x)$

86 If $f(x) = 8x + 4$ the $f^{-1}(x) = ?$

a. 1/(8x+4) b. (x-4)/8 c. (8x+4)/(4-8x) d. None of these

$y = f(x) = 8x + 4$

$\therefore x = \frac{y-4}{8} = f^{-1}(y)$

$f^{-1}(x) = \left(\frac{x-4}{8}\right)$

87 If $h(x) = \left(\frac{px-q}{qx-p}\right)$ then x = ?

a. h (1/y) b. h(-y) c. h (y) d. None of these

$\Rightarrow y = h(x) = \frac{px-q}{qx-p}$

$qxy - py = px - q$

$qxy - px = py - q$

$x(qy - p) = py - q$

$x = \left(\frac{py-q}{qy-p}\right)$ ..... (1)

$h(x) = \left(\frac{px-q}{qx-p}\right)$

$h(y) = \left(\frac{py-q}{qy-p}\right)$ ..... (2)

As per each (1) & (2)

$x = h(y)$

My Notes

88 A set of intelligent students in a class is _____

a. Null set

b. Singleton set

c. An infinite set

~~d.~~ Not a well defined collection (As intelligence is a relative concept)

89 If $f(x+1) = f(x-1)$ where $f(x) = x^2 - 2x + 3$ then x = ?

~~a.~~ 1 b. 2 c. 3 d. None of these

$f(x+1) = f(x-1)$

$(x+1)^2 - 2(x+1) + 3 = (x-1)^2 - 2(x-1) + 3$

$x^2 + 2x + 1 - 2x - 2 = x^2 - 2x + 1 - 2x + 2$

$\therefore 4x = 4$

$x = 1$

90 If $f(x+1) = f(x+2)$ where $f(x) = 1 + x - x^2$ then x = ?

a. 2 b. 0 c. 1 ~~d.~~ -1

$f(x+1) = f(x+2)$

$1 + x + 1 - (x+1)^2 = 1 + x + 2 - (x+2)^2$

$1 - x^2 - 2x - 1 = 2 - x^2 - 4x - 4$

$-2x = -4x - 2$

$2x = -2$

$\boxed{x = -1}$

91 If $f(x) = 3x + 4$ then $f[(x-4)/3]$ = ?

a. 1 ~~b.~~ x c. zero d. None of these

$\Longrightarrow f\left(\frac{x-4}{3}\right) = 3 \times \frac{(x-4)}{3} + 4$

$= x - 4 + 4 = x$

92 If $f(x+1) = 4x + 5$; find $f(x)$

a. $3x+4$ ~~b.~~ $4x+1$ c. $4x+3$ d. None of these

$f(x+1) = 4x + 5$

$f(x-1+1) = 4(x-1) + 5 = 4x - 4 + 5 = 4x + 1$

$f(x) = 4x + 1$

93 If $f(x-1) = x^3$; find $f(x)$

~~a.~~ $(x+1)^3$ b. $(x+1)^2$ c. x^3 d. $(x-1)^3$

$f(x-1) = x^3$

$f(x+1-1) = (x+1)^3 \quad \therefore f(x) = (x+1)^3$

94 $f(x) = 3x + 5$; $g(x) = 6x + 100$. Find $g[f(2x)]$ = ?

a. $16x + 200$ b. $9x - 300$ c. $f(x)$ ~~d.~~ None of these

$\Longrightarrow g \cdot f(2x) = g[3(2x) + 5] = g(6x + 5)$

$= 6(6x + 5) + 100$

$= 36x + 130$

95 If S = {0,1,5,4,7,9,10} then $\therefore n(S) = 7$

No. of subsets = $2^7 = 128$

No. of proper subsets = $2^7 - 1 = 127$

No. of non empty subsets = $2^7 - 1 = 127$

No. of non empty proper subsets = $2^7 - 2 = 126$

96 If $A \subseteq B$ then

a. $A' \subseteq B'$ b. $A' = B'$ ~~c. $B' \subseteq A'$~~ d. None of these

$A = \{1,2,3\}$

$B = \{1,2,3,4,5\}$

$U = \{1,2,3,4,5,6,7,8\}$

$A' = \{4,5,6,7,8\}$

$B' = \{6,7,8\}$

When $A \subseteq B$ then $B' \subseteq A'$

97 If 'A' is any set then

a. $A \cup A' = \phi$ ~~b. $A \cap A' = \phi$~~ c. $A \cup \phi = A'$ d. None

98 f(x-1) = 2x-2 then f(16) is

a. 16 b. 15 ~~c. 32~~ d. Insufficient information

$f(x-1) = 2x - 2$

$f(x-1) = 2(x-1)$

$f(p) = 2p$

$\therefore f(16) = 2 \times 16 = 32$

99 If A = {1,2,3,5,7}, B = {1,3,6,10,15} and universal set = U = {1,2,3,4,5,.....,15} then cardinal value of

$(A \cap B) = 2$

$(A \cup B) = 8$

$(A - B) = 3$

$(B - A) = 3$

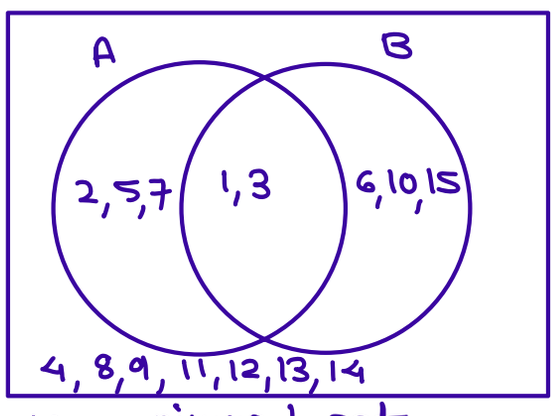
$(A' \cap B') = 7$

$(A \triangle B) = 6$

$(A \cup B') = 12$

$(B \cup A') = 12$

$(A' \cup B') = 13$



U = universal set

$n(U) = 15$

100 Null set don't have a proper subset

~~a. True~~ b. False

Null set has only one subset i.e. Null set only. which is an improper subset.

101 Find All subsets of A = {5,8,9,12}

{5}, {8}, {9}, {12}, {5,8}, {5,9}, {5,12}, {8,9}, {8,12}, {9,12}, {5,8,9}, {5,8,12}, {5,9,12}, {8,9,12}, { }

proper subsets

Improper subset

102 Find power set of A if A = {2,8,9}

power set of A = { {2,3}, {8,3}, {9,3}, {3,8,3}, {3,9,3}, {8,9}, { }, {2,8,9} }

Set of all possible subsets is known as power set

103 If universal set U = {1,2,3,4,5,.....,25}; A= {2,6,8,10,12,.....,24}
B = {4,8,10,14} then

a. (A ∩ B)' = (A' ∪ B') b. (A∩B)' = A'∩B' c. A'∩ B' = A' d. (A' ∪ B') = A'

104 P set has 3 elements, Q set has 4 elements then the set (PxQ) contains _____ elements

a. 34 b. 7 c. 1 d. 12

$n(P \times Q) = n(P) \times n(Q)$

$= 3 \times 4 = 12$

105 If $f(x) = 2^x$ then function is

a. one-one b. one-many c. many-one d. many-many

106 If $f(x) = e^x$ then f(p-q) is

a. f(p) + f(q) b. f(p) - f(q) c. f(p) x f(q) d. f(p) / f(q)

$f(x) = e^x$

$f(p) = e^p$

$f(q) = e^q$

$f(p-q) = e^{p-q} = \frac{e^p}{e^q} = \frac{f(p)}{f(q)}$

My Notes

If $f(x) = x^2 + x^3$ then $f(x)$ is

a) odd function b) Even function c) neither odd nor even d) a & b

$f(x) = x^2 + x^3$

$f(-x) = (-x)^2 + (-x)^3 = x^2 - x^3$

$f(2) = 2^2 + 2^3 = 12$

$f(-2) = (-2)^2 + (-2)^3 = 4 - 8 = -4$

107 If A = {x : x < 1 and x > 1} then set A is

~~a.~~ Null set b. Singleton set c. Infinite set d. Power set

$A = \{\ \}$

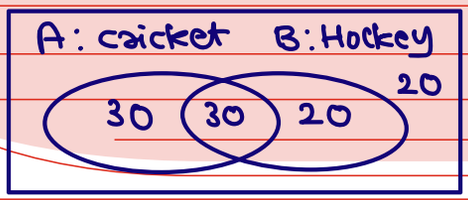
108 Set of Even Prime natural numbers is

a. Null set ~~b.~~ Singleton set c. Infinite set d. Power set

Set of Even prime natural numbers $= \{2\}$

109 In a class of 100 students 60 play Cricket, 50 play Hockey and 30 play both. Then no. of students who don't play atleast one of 2 games is :

~~a.~~ 70 b. 50 c. 10 d. None of these



$n(A' \cup B') = 70$

$n(A' \cup B') = n(A') + n(B') - n(A' \cap B')$

$= 40 + 50 - 20 = 70$

110 If $f(x) = (x+1)/(x-1)$; then $f^{-1}(30) = ?$

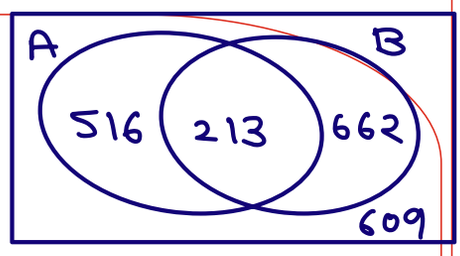
a. 23/12 b. 30/8 ~~c.~~ 31/29 d. None of these

$\Rightarrow y = f(x) = \frac{x+1}{x-1}$ $\therefore x = \left(\frac{y+1}{y-1}\right)$ $f^{-1}(30) = \frac{30+1}{30-1}$

$xy - y = x + 1$ $f^{-1}(y) = \frac{y+1}{y-1}$ $= 31/29$

$xy - x = y + 1$

111 n(A) = 729, n(B) = 875, n(A∩B) = 213, n(U) = 2000. Find -



n(A') = 1271

n(B') = 1125

n(A - B) = 516

n(B - A) = 662

n(A'∩B') = 609

n(A △ B) = 1178

n(A'∪ B') = 1787

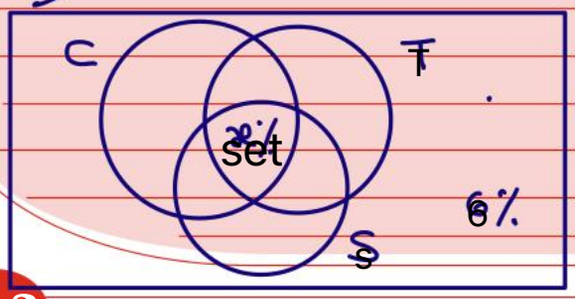
n(A ∪ B') = 1338

n(B ∪ A') = 1484

My Notes

112 Out of 2000 employees in an office 48% preferred coffee (C) and 54% liked Tea (T) and 64% used to smoke (S) 28% used C & T. 32% used T & S. 30% preferred C & S. Only 6% did none of these. The number having all three is :

a. 360 b. 300 c. 380 d. None of these



$n(C \cup T \cup S) = 94\%$

$0.48 + 0.54 + 0.64 - 0.28 - 0.32 - 0.30 + x = 0.94$

$0.76 + x = 0.94$

$x = 18\%$

Number having all three $= 18\%\ (2000) = 360$

113 P set has 11 elements & Q set has 12 elements then (P x Q) has _____ elements

a. 1 b. 23 c. 132 d. 11/12

$n(P \times Q) = n(P) \times n(Q) = 11 \times 12 = 132$

114 If A = {5,7,8} B = {7,5,8} show that (AxB) = (BxA) = (AxB) ∪ (BxA) = (AxB) ∩ (BxA)

$A \times B = \{(5,7), (5,5), (5,8), (7,7), (7,5), (7,8), (8,7), (8,5), (8,8)\}$

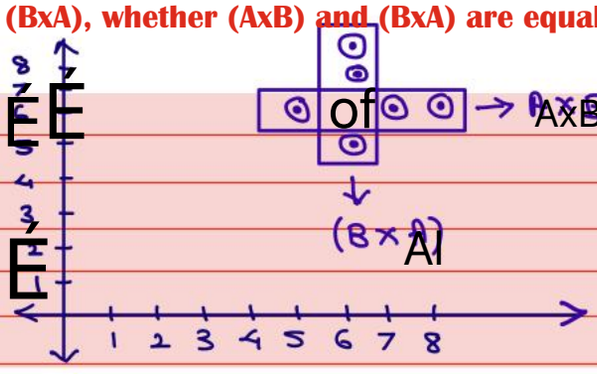
$= (B \times A) = (A \times B) \cup (B \times A) = (A \times B) \cap (B \times A)$

Therefore, (A x B) and (B x A) are equal as well as equivalent sets are.

115 If A = {5,7,8} B = {6}; Find (AxB), (BxA), whether (AxB) and (BxA) are equal sets? Equivalent sets?

$A \times B = \{(5,6), (7,6), (8,6)\}$

$B \times A = \{(6,5), (6,7), (6,8)\}$



(AxB) & (BxA) are Equivalent sets but NOT equal sets that sets

116 In (AxB) = {(a,b) : where a ∈ A , b ∈ B }

If A = ϕ or B = ϕ then we defined (AxB) or (BxA) as ϕ

A x B is a set of all ordered pairs (i, j) where i ∈ A & j ∈ B

My Notes

(BxA) is a set of all ordered pairs (x, y) where x ∈ B and y ∈ A

117 If A = {12,10,16} B = {5,8,12,13} C = {8,11,10,25,16}

Find a. A x (B ∩ C)

b. B x (A ∩ C)

① $A \times (B \cap C) = \{12,10,16\} \times \{8\} = \{(12,8),(10,8),(16,8)\}$

② $B \times (A \cap C) = \{5,8,12,13\} \times \{10,16\}$

$= \{(5,10),(8,10),(12,10),(13,10),(5,16),(8,16),(12,16),(13,16)\}$

118 If f(x) = (x+1) / (x-1). Find f(-3/2), f(7/3)

$f(x) = \left(\frac{x+1}{x-1}\right)$, $f\left(-\frac{3}{2}\right) = \frac{-\frac{3}{2}+1}{-\frac{3}{2}-1} = \frac{-3+2}{-3-2} = \frac{-1}{-5} = \left(\frac{1}{5}\right)$

$f\left(\frac{7}{3}\right) = \left(\frac{\frac{7}{3}+1}{\frac{7}{3}-1}\right) = \frac{7+3}{7-3} = \frac{10}{4} = \left(\frac{5}{2}\right)$

119 If $g(x) = \left(\frac{2x+1}{3x+8}\right)$; f(x) = 8x + 5; Find f.g(10); g.f(-2); g[f⁻¹(5)]

⟹ ① $f \cdot g(10) = f\left[\frac{2(10)+1}{3(10)+8}\right] = f\left(\frac{21}{38}\right)$

$= 8\left(\frac{21}{38}\right) + 5 = \frac{168}{38} + 5 = \frac{358}{38}$

$= \left(\frac{179}{19}\right)$

② $g \cdot f(-2) = g\left[8(-2)+5\right] = g(-11)$

$= \left[\frac{2(-11)+1}{3(-11)+8}\right] = \frac{-21}{-25} = 21/25$

③ $y = f(x) = 8x + 5$ $\therefore x = \frac{y-5}{8} = f^{-1}(y)$

$f^{-1}(5) = \frac{5-5}{8} = 0$

$g\left[f^{-1}(5)\right] = g(0) = \frac{2(0)+1}{3(0)+8} = 1/8$

My Notes

If $f(x) = 3x + 13$, $g(x) = 10x + 21$. Find $g\left[f^{-1}(12)\right]$

⟹ $y = f(x) = 3x + 13$

$f^{-1}(y)$ $x = \frac{y-13}{3}$

$f^{-1}(12) = \frac{12-13}{3} = -1/3$

$g\left[f^{-1}(12)\right] = g(-1/3)$

$= 10\left(-\frac{1}{3}\right) + 21$

$= 21 - \frac{10}{3} = \left(\frac{53}{3}\right)$

120 f(x) = 1/(1-x). Find f(-1)

a. 1 ~~b. 1/2~~ c. Not defined d. 2

$f(x) = \frac{1}{1-x}$

$f(-1) = \frac{1}{1-(-1)} = \frac{1}{2}$

121 {(x,y) : x < y and x,y ∈ R} is

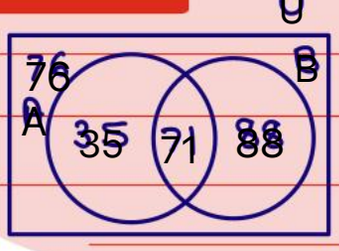
~~a.~~ not a function b. a function c. one-one mapping d. None of these

A = {(5,7), (5,8), (5,10), (5, 33), (2,50) ..... }

122

1. A∪A = A
2. A ∪ A' = U
3. A ∩ A'= ϕ
4. A ∪ U = U
5. A ∪ Φ = A
6. A ∩ Φ = ϕ
7. Φ ∪ A' = A'
8. Φ ∩ U = ϕ
9. (A-B)∩(B-A) = ϕ
10. (A∪B) ∪ (A∩B) = (A∪B)
11. (A∪B) ∩ (A∩B) = (A∩B)
12. (A∪B) ∪ A = (A∪B)
13. (A∪B) ∩ A = A
14. (A∩B) ∪ A = A
15. (A∩B) ∩ A = (A∩B)
16. (A ∪ B) ∪ A' = U
17. (A ∪ B) ∩ (A'∩B') = ϕ
18. (A△B) ∪ (A∩B) = (A∪B)
19. (A'∪ B') ∪ (A∩B) = U
20. (A-B) ∪ (A∩B) = A
21. (B-A) ∪ B = B
22. (A△B) ∪ (A'∩B') = (A∩B)' = (A' ∪ B')
23. (A'∩B') ∪ (A△B) = (A∩B)' = (A' ∪ B')
24. (A△B) ∪ A = (A∪B)

My Notes

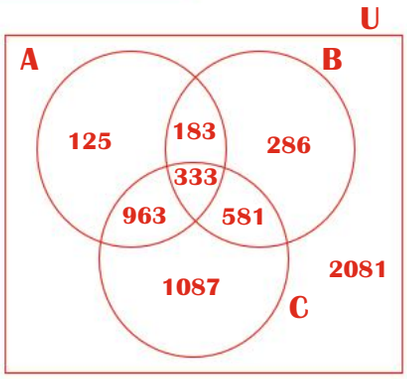


n[(A△B) ∪ A] = n(A∪B) = 194

n[(A∪B) ∪ (A'∩B')] = n(U) = 270

n(A∩A') = 0

123



1. n(A) = 1604
2. n(B) = 1383
3. n(C) = 2964
4. n(A') = 4035
5. n(B') = 4256
6. n(C') = 2675
7. n(U) = 5639
8. n(A ∩ B) = 516
9. n(B ∩ C) = 914
10. n(A ∩ C) = 1296
11. n(A ∪ B) = 2471
12. n(B ∪ C) = 3433
13. n(A ∪ C) = 3272
14. n(A-B) = 1088
15. n(B-A) = 867
16. n(A-C) = 308
17. n(C-A) = 1668
18. n(B-C) = 469
19. n(C-B) = 2050
20. n(A' ∩ B') = 3168
21. n(B' ∩ C') = 2206
22. n(A' ∩ C') = 2367
23. n(A △ B) = 1955
24. n(B △ C) = 2519
25. n(A △ C) = 1976
26. n(A ∪ B') = 4772
27. n(B ∪ A') = 4551
28. n(A ∪ C') = 3971
29. n(C ∪ A') = 5331
30. n(B ∪ C') = 3589
31. n(C ∪ B') = 5170
32. n(A' ∪ B') = 5123
33. n(B' ∪ C') = 4725
34. n(A' ∪ C') = 4343
35. n(A ∪ B ∪ C) = 3558
36. n(A ∩ B ∩ C) = 333
37. n(A' ∩ B' ∩ C') = 2081
38. n(A ∩ B' ∩ C') = 125
39. n(A' ∩ B ∩ C') = 286
40. n(C ∩ A' ∩ B') = 1087

My Notes

If n(A) = 5281, n(B) = 7863, n(A ∩ B) = 4448

Find n(A △ B), n(A ∪ B')

$\Rightarrow n(A \triangle B) = n(A \cup B) - n(A \cap B)$

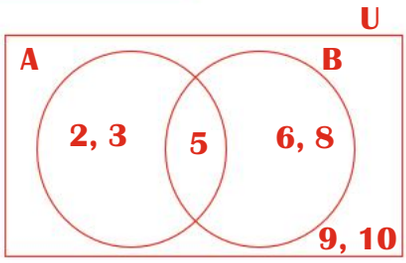
$= n(A) + n(B) - n(A \cap B) - n(A \cap B)$

$= 5281 + 7863 - 4448 - 4448 = 4248$

$n(A \cup B') = n(U) - n(B - A)$

$=$ Insuff. data

124



Find Sets :

1. A = { 2,3 }
2. B = { 5,6,8 }
3. (A ∩ B) = { 5 }
4. (A ∪ B) = { 2,3,5,6,8 }
5. (A ∩ B') = { 2,3 }
6. (B ∩ A') = { 6,8 }
7. (A' ∩ B') = { 9,10 }
8. (A △ B) = { 2,3,6,8 }
9. (A ∪ B') = { 2,3,5,9,10 }
10. (B ∪ A') = { 5,6,8,9,10 }
11. (A' ∪ B') = { 2,3,6,8,9,10 }

125

B = {8,9,3,6,8,9,6,6,8,9,11,13,8,9,9,15}

Cardinal Value of Set B is 7.

B = { 8,9,3,6,11,13,15 }

As there are 7 distinct observations in set B.